

# Thermohydrodynamic Characteristics of Journal Bearings Lubricated with Ferrofluid

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**Abstract**-In this work the thermohydrodynamic (THD) characteristics of journal bearings lubricated with ferrofluid is investigated. Based on the momentum and continuity equation a pressure differential equation was obtained taking into consideration the dependence of the lubricant viscosity on the temperature. Assuming linear behavior for the magnetic material of the ferrofluid the magnetic pressure was incorporated into the Reynolds equation. In this treatment the shaft is taken to be isothermal and the bearing conducts heat in the radial direction, this last assumption reduced the iteration scheme and the solution required the satisfaction of the Reynolds equation and the energy equation together with their boundary conditions to arrive at lubricant temperature and pressure fields. Results showed that thermal effect should be taken into consideration in the design of journal bearings operating at higher values of eccentricity ratio whether the bearing is using conventional lubricant or ferrofluid one. Also it is confirmed that when compared with conventional lubricants, ferrofluids enhance the bearing performance characteristics especially when taking into consideration thermal effects.

## 1. INTRODUCTION

Ferro fluids are an interesting group of liquids, because they have liquid properties and act like a ferromagnetic material. Many properties of the Ferro fluid are similar to the base fluid. Among the various applications in engineering, are those taking the advantage of introducing an external force into the fluid to facilitate control, position, orientation and movement of the fluid. The most usual applications are in sealing, filtering, separation, and ink-jet printing [1-3]. Magnetic fluids have also been used in the lubrication of journal bearings with some advantages over conventional lubrication.

Tiepi [4, 5] calculated analytically the bearing performance characteristics for infinitely long and short bearings lubricated with ferrofluid without calculating the magnetic force from the assumed magnetic field distribution. A full

analytical investigation has been introduced by Osman and et al. [7] in which a pressure differential equation was obtained based on the momentum and continuity equations. Assuming linear behavior for the magnetic material of the ferrofluid, the magnetic force was calculated. The magnetic pressure resulting from the magnetic force was incorporated into the Reynolds equation and it was not separately treated. The Solution rendered the bearing performance characteristics, namely, load carrying capacity, attitude angle, frictional force at the journal surface, and bearing side leakage. Recently Huang and et al.[8] showed that  $Fe_3O_4$ -based Ferro fluids under a magnet has a higher supporting capacity compared with the carrier liquid, also significant improvement in antifriction and wear resistance was obtained..

It must be noted that the bearing performance characteristics of ferrofluid

journal bearing are affected by the viscosity of the lubricant which is in turn affected by the temperature and the applied magnetic field. The effect of the magnetic field has been investigated by many authors, and it was concluded by Osman and et al. [9] that the bearing performance is significantly modified when the magnetic effects are comparable with the hydrodynamic ones, namely; when the bearing operates at relatively low eccentricity ratio, high magnetic field and low rotational speed.

Because of the strong dependence of lubricant viscosity on temperature this requires knowledge of the temperature field in the lubricant. Thermal effects have been studied extensively for laminar and turbulent flow journal bearings and the first complete thermohydrodynamic solution was introduced by McCall ion [10] with an effective viscosity taken at the mean average temperature of the lubricant. Ezzat et al. [11] confirmed that an effective viscosity may overestimate the load carrying capacity of the bearing depending upon the oil inlet temperature and the speed of operation. A complete analysis of this problem requires the simultaneous solution of the momentum and energy equations for the lubricant and the equation of heat conduction for the bearing and the journal. Safar [12] introduced a full thermohydrodynamic solution for laminar flow journal bearing assuming that the viscosity is dependent on the temperature, in the analysis the shaft was considered isothermal and the bearing conducts heat in the radial direction. The last assumption led to essential simplification in which the bearing was replaced by an equivalent homogenous boundary condition for the energy equation and good results were obtained.

Thermal effects of laminar flow journal bearings have been investigated by many authors taking into consideration

other geometries or parameters. Singh et al. [13], and others presented thermohydrodynamic analysis for journal bearings with axial grooves, Chauhan et al. [14] considered elliptical journal bearings. Thermal effects were considered for misaligned journal bearings by Safar et al. [15] Jun Sun et al. [16] and others, and it was concluded that isothermal solution overestimates bearing performance characteristics especially at higher values of eccentricity ratio and degree of misalignment.

Thermal effects were considered for laminar flow journal bearings lubricated with non Newtonian fluid by J F Lin et al. [17] and Ju Shean Ming et al. [18], and others. It was concluded that thermal effects are found to be more pronounced at higher eccentricity and slenderness ratios.

From previous work it is found that it is essential to include thermal effects in the design of laminar flow journal bearing since in some circumstances the bearing performance characteristics are substantially changed.

To date thermal effects were not considered in laminar flow journal bearings lubricated with ferrofluids. The present paper presents a thermohydrodynamic solution for a laminar flow journal bearing lubricated with ferrofluids and compares their characteristics with conventional ones.

## 2. ANALYSIS

For a ferrofluid under a magnetic field, the unit volume value of the induced magnetic force is given by  $f_m = \mu_o X_m \hat{j}_m \text{ grad } \hat{j}_m$

Consistent with the basic assumption of creeping flow and the type of geometry which is particular to lubricating films, and using the magnetic force as a body external force, the equation of motion for the fluid film are derived as follow

$$\begin{aligned}
 0 &= -\frac{\partial P}{\partial X} + \frac{\partial}{\partial X} \left\{ \eta \frac{\partial u}{\partial X} \right\} + f_{mx} \\
 0 &= -\frac{\partial P}{\partial Z} + \frac{\partial}{\partial X} \left\{ \eta \frac{\partial w}{\partial X} \right\} + f_{mz}
 \end{aligned}
 \tag{1}$$

In equation (1)  $\eta = \eta(T)$ , thus the solution of (1) presuppose the knowledge of the temperature field in the oil film.

Integration of equation (1) with respect to  $y$  and substitution of the resulting velocity components into the equation of continuity leads to generalized pressure equation with variable viscosity. The dimensionless modified Reynolds equation for a ferrofluid lubricant taking into consideration thermal effect becomes

$$\begin{aligned}
 \frac{\partial}{\partial \theta} \left\{ H^3 \Gamma(\theta) \frac{\partial p}{\partial \theta} \right\} + \frac{1}{4} \frac{\partial}{\partial z} \left\{ H^3 \Gamma''(\theta) \frac{\partial p}{\partial z} \right\} \\
 2\pi \left[ \frac{d}{d\theta} \left\{ H \frac{I_1(\theta)}{I_0(\theta)} + \alpha \frac{\partial}{\partial z} \left( H^3 j_m(\theta) \frac{\partial j_m}{\partial z} \right) \right\} \right]
 \end{aligned}
 \tag{2}$$

where

$$\begin{aligned}
 \Gamma(\theta) &= I_2(\theta) - \frac{I_1(\theta)}{I_0(\theta)} \\
 I_0(\theta) &= \int_0^1 \frac{dy}{\xi(\theta, y)} \\
 I_1(\theta) &= \int_0^1 \frac{y dy}{\xi(\theta, y)}
 \end{aligned}$$

$$I_2(\theta) = \int_0^1 \frac{y^2 dy}{\xi(\theta, y)} \tag{3}$$

The RHS of equation (2) contains the wedge action effect and the magnetic effect due to field gradient in the axial direction only since it was show by Osman and el al. [9] that the axial symmetric magnetic field which has a gradient only in the axial direction can enhance significantly the bearing performance characteristics namely load carrying capacity and side leakage.

Therefore the axial parabolic distribution magnetic field is used. It is represented by the following equation

$$j_m(Z) = (j_{mc} - j_{me}) \left( \frac{2Z}{L} \right)^2 \tag{4}$$

In non dimensional form, it is given by

$$j_m(z) = 1.0 - 4(1 - \beta)z^2 \tag{5}$$

Where  $\hat{j}_m = j_m / j_{mc}$  and  $z = Z/L$ .

$\beta$  is the ratio of the magnetic field strength at end section ( $h_{me}$ ) to its value at the middle section ( $h_{mc}$ ). It is an important parameter that determines the gradient of the magnetic field. Negative magnetic gradient is required to obtain positive induced magnetic pressures and the resultant load carrying capacity will then be increased. This can be achieved for values of  $\beta$  ranging from 0 to less than 1. If  $\beta = 1.0$ , there is no field variation ( $\partial \hat{j}_m / \partial z = 0.0$ ) and equation (2) will be turned into the Reynolds equation and the bearing will then perform as a conventional bearing. On the other hand, if the magnetic field variation is such that  $\partial \hat{j}_m / \partial z$  is positive (for  $\beta > 1.0$ ), negative magnetic pressures will be induced and the bearing

performance characteristics is then decreased.

It was shown by Osman and et al. [7] that for  $\beta=0.75$  all bearing characteristics are improved and the maximum sealing effect is obtained for  $\beta=0.5$ .

In this paper  $\beta$  is chosen to be 0.75 to have the highest bearing load. Also another important parameter in the magnetic field which represents the strength of the magnetic effect is the magnetic force coefficient  $\alpha = (h_{m0})^2 \mu_0 X_m c^2 / \eta \omega L^2$

Here two values for  $\alpha$  is used,  $\alpha=0$  which it represents a conventional bearing and  $\alpha=0.2$

To solve equation (2) the viscosity distribution must be known. The lubricant viscosity is calculated by

$$\xi(\theta, y) = e^{-g t(\theta, y)} \quad (6)$$

where  $t$  is the temperature of the lubricant and  $g$  is a constant to be chosen for the particular oil

In deriving equation (2) the density was taken constant and the viscosity is variable and dependent on the temperature of the lubricant only. The boundary conditions of equation (2) are

$$\begin{aligned} p\left(\theta, \pm \frac{L}{2D}\right) &= 0 \\ p(\theta, z) &= 0 \\ p(\theta_2, z) &= \frac{\partial p(\theta_2, z)}{\partial \theta} = 0 \end{aligned} \quad (7)$$

The third of conditions (7) establishes the position of the trailing edge

The dimensionless energy equation of the lubricant, and for a laminar flow bearing as was given by Safar [12] as

$$H^3 u \frac{\partial t}{\partial \theta} = \frac{1}{P_e} \frac{\partial t}{\partial y^2} + \xi \left\{ \frac{\partial u}{\partial y} \right\}^2 \quad (8)$$

The boundary conditions associated with energy equations are

$$\begin{aligned} t(0, y) &= t_i \\ t(\theta, 1) &= t_s \\ \left. \frac{-k_o}{cH} \frac{\partial t}{\partial y} \right]_{y=0} &= \left. k_b \frac{\partial t_b}{\partial r} \right]_{r=R_b} \end{aligned} \quad (9)$$

$$t(\theta, 0) = t(\theta, R_b)$$

In order to solve equation (8) together with the boundary conditions specified in equation (9), the heat conduction equation in the bearing must be solved. Neglecting circumferential heat flow in the bearing in comparison with radial conduction will lead to significant simplification as was done by Safar [12], and the last conditions of equation (9) together with the heat conduction equation in the bearing will yield a relationship

$$\begin{aligned} \left\{ t + \gamma(\theta) \frac{\partial t}{\partial y} \right\}_{y=0} &= 0 \\ \gamma(\theta) &= - \frac{1}{H(\theta)} \left( \frac{R}{c} \right) \left\{ \frac{1}{N_u} + \frac{k_o}{k_b} \ln \left( 1 + \frac{b}{R} \right) \right\} \end{aligned} \quad (10)$$

Also it is assumed that the shaft temperature will be equal to the average temperature of the bearing.

### 3. SOLUTION

In order to arrive at lubricant temperature and pressure fields one is required to satisfy simultaneously equations (2),(8) and (10) simultaneously with their boundary conditions, therefore an arbitrary viscosity field is assumed to

integrate equations (3), hence equation (2) is solved numerically.

Velocity components and its gradients are calculated, consequently the temperature distribution is obtained for equation (6), and the previous steps are repeated till convergence of the viscosity distribution is obtained.

Integration of the pressure over the bearing area gives the non-dimensional load carrying capacity, calculated by

$$w = \sqrt{w_\varepsilon^2 + w_\phi^2} \quad (11)$$

$$w_\varepsilon = 2 \int_0^{0.5} \int_0^{2\pi} p \cos \theta d\theta dz$$

$$w_\phi = 2 \int_0^{0.5} \int_0^{2\pi} p \sin \theta d\theta dz$$

The attitude angle,  $\phi$ , is calculated by

$$\phi = \tan^{-1} \frac{w_\phi}{w_\varepsilon} \quad (12)$$

The non-dimensional frictional force at the journal surface can be given by

$$f = 2 \int_0^{0.5} \int_0^{2\pi} \left\{ 0.5 \frac{\partial p}{\partial \theta} H + \frac{k}{H} \right\} d\theta dz \quad (13)$$

where  $f = F \{ (c/R)^2 / \eta \omega L c \}$  and  $k$  is a control number. Inside the active zone region (full film thickness),  $k=1.0$  and outside this region (partial film thickness), it is calculated by  $k = h_{\min} / h < 1.0$ ; where  $h_{\min}$  is the minimum film thickness.

The side leakage can be obtained by integrating the axial velocity component across the end section, It is calculated by

$$q = \int_0^{2\pi} \frac{H^3}{6} \left[ \left( \frac{1}{4(L/D)^2} \frac{\partial p}{\partial z} \xi H_m \frac{\partial H_m}{\partial z} \right) \right]_{z=0} \quad (14)$$

#### 4. RESULTS AND DISCUSSION

Solutions of dimensionless pressure and temperature were obtained for a magnetized laminar flow journal bearing for various values of  $\varepsilon$ . The dimensionless load frictional force and, lubricant side leakage have been obtained for a bearing with a length to diameter ratio of unity ( $L/D=1$ ) with oil inlet temperature equals to the reference temperature.

The results obtained for the thermohydrodynamic case are for Pecklet number ( $Pe = \rho c_p \omega c^2 / k_0$ ) equal to 200.

Figures (1a, 1b, 1c) display the non dimensional centerline pressure distribution in the circumferential direction at eccentricity ratio of 0.1 for  $\alpha=0, 0.1$  and  $0.2$  respectively. For  $\alpha=0$  it represents the conventional bearing, and as seen from the figure that the isothermal solution overestimates the maximum pressure by 50%, and as  $\alpha$  increases, thermal effects are less pronounced. The isothermal solution overestimates the maximum pressure by 33% and 24% for  $\alpha=0.1$  and  $0.2$  respectively. This shows that those thermal effects are less pronounced as the magnetic force coefficient increases.

Figure (1b)  
 Nondimensional centerline pressure distribution  
 for  $\alpha=0.1$  and eccentricity ratio 0.1

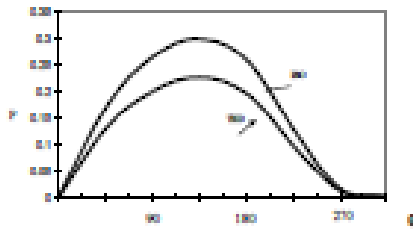


Figure (1a)  
 Nondimensional centerline pressure distribution  
 for  $\alpha=0$  and eccentricity ratio 0.1

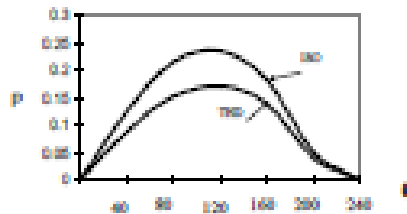


Figure (1c)  
 Nondimensional centerline pressure distribution  
 for  $\alpha=0.2$  and eccentricity ratio 0.1

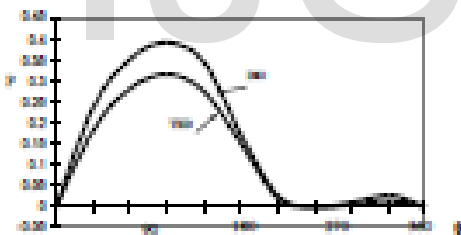


Figure (2b)  
 Nondimensional load capacity versus  $\alpha$   
 for  $\alpha=0.2$

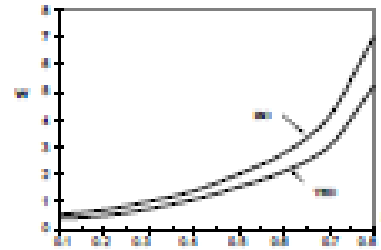
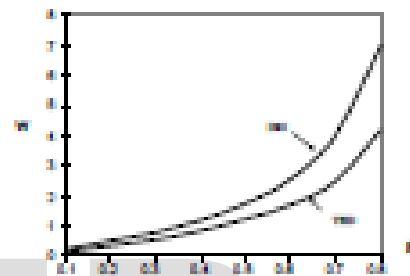


Figure (2a)  
 Nondimensional load capacity versus  $\alpha$   
 for  $\alpha=0.0$



The variation of the nondimensional friction force with the eccentricity ratio is shown in figures (3a, 3b) for  $\alpha=0$  and 0.2 respectively for both isothermal and thermohydrodynamic solutions. It is clear that thermal effects are more pronounced at higher values of eccentricity ratio and the effect is more evident for journal bearings lubricated with conventional lubricants than those lubricated with ferrofluid.

Figures (2a, 2b) show the variation of the nondimensional load capacity with the eccentricity ratio for the cases when  $\alpha=0$ , i.e., the conventional bearing, and for  $\alpha=0.2$ , i.e., lubricated with the ferrofluid. In each figure both the isothermal and the thermohydrodynamic solutions are presented. It is noted that isothermal solution overestimates the load carrying capacity at eccentricity ratio of 0.8 by 70% and 40% for  $\alpha=0$  and 0.2 respectively.

Figure (4a)  
 Nondimensional friction force versus  $\epsilon$   
 for  $\alpha = 0$

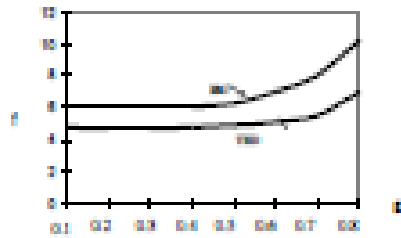


Figure (4b)  
 Nondimensional side leakage versus  $\epsilon$   
 for  $\alpha = 0$

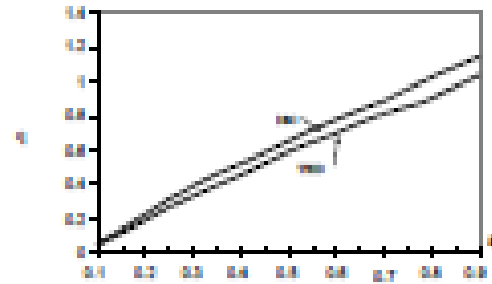


Figure (4c)  
 Nondimensional friction force versus  $\epsilon$   
 for  $\alpha = 0.2$

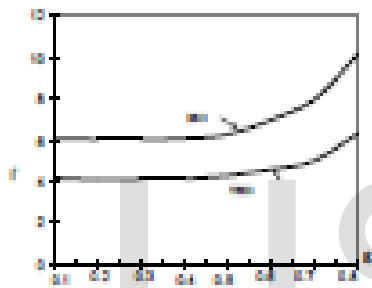
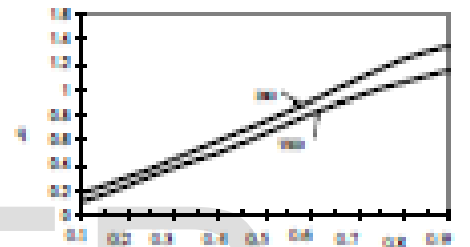


Figure (4d)  
 Nondimensional side leakage versus  $\epsilon$   
 for  $\alpha = 0.2$



Figures (4a, 4b) show the variation of the nondimensional side leakage with the eccentricity ratio. It is shown that there is considerable decrease in the side leakage for journal bearings lubricated with ferrofluids and that decrease is more pronounced at smaller values of eccentricity ratio. When taking into consideration thermal effects it is observed the decrease in the side leakage is almost the same for both bearings lubricated with conventional and ferrofluid lubricants.

## 5. CONCLUSIONS

The results confirm that ferrofluids improve the hydrodynamic characteristics of journal bearing and provides higher load capacity and reduced frictional force and lubricant side leakage.

It is evident from the results that thermal effect should be taken into consideration in the design of journal bearings operating at higher values of eccentricity ratio whether the bearing is using conventional lubricant or ferrofluid one.

Results indicated that thermal effects are less pronounced on the bearing performance characteristics when lubricated with ferrofluids, which concludes that ferrofluids as a lubricant have better

thermal characteristics than conventional ones. Also the results confirmed that when compared with conventional lubricants, ferrofluids enhance the bearing performance characteristics specially when taking into consideration thermal effects

## 6. REFERENCES

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Nomenclature

b	radial dimension of bearing
c	bearing clearance
$c_p$	lubricant specific heat
D	bearing diameter
e	eccentricity of the journal center
$F_j$	friction force at the journal surface
F	dimensionless friction force
=	$F_j (c/R)^2 / (\eta \omega L c)$
$f_m$	unit volume value of the induced magnetic force
h	lubricant film thickness (= $c (1 + \varepsilon \cos \theta)$ )
H	dimensionless film thickness $H = h/c$
$h_m$	magnetic field intensity
$h_{m0}$	characteristic value of magnetic field intensity
$H_m$	dimensionless of magnetic field intensity $H_m = h_m/h_0$
I	strength of the current passing through the wire
K	distance ratio parameter $K = R_0 / R$
$k_o$	lubricant heat conductivity
$k_b$	bearing heat conductivity
L	bearing length
Nu	Nusselt number (= $\lambda(R+b)/k_o$ )
$M_g$	magnetization of the ferrofluid
$\rho$	dimensionless pressure $\rho = P(c/R)^2 / \eta_i \omega$
Pr	Prandtl number (= $c_p \eta / k_o$ )
P	lubricant pressure
$P_e$	Pecklet number = $\rho c_p \omega c^2 / k_o$
q	dimensionless side leakage = $2Q/LRc\omega$
Q	bearing side leakage
R	bearing or journal radius
$R_0$	displaced distance from the wire position to bearing center
T	temperature
$T_i$	reference temperature
t	dimensionless lubricant temp (= $c_p \rho (T - T_i) / (\eta_i \omega (R/c)^2)$ )
$U_x$	circumferential velocity component
$U_\omega$	axial velocity component
w	dimensionless load-carrying capacity $w = \frac{W(c/R)^2}{\eta \omega LR}$

W	load-carrying capacity
$w_\varepsilon$	dimensionless load capacity in the eccentricity direction
$w_\theta$	dimensionless load capacity normal to the eccentricity line
$X_m$	susceptibility of ferrofluid
X, Y, Z	Cartesian coordinates
y	dimensionless distance from the bearing (= $y/h$ )
z	dimensionless axial distance Z/L
$\alpha$	magnetic force coefficient $\frac{(h_{m0})^2 \mu_0 X_m c^2}{\eta \omega L^2}$
$\varepsilon$	eccentricity ratio $\varepsilon = e/c$
$\theta$	attitude angle
$\eta$	fluid viscosity
$\theta$	angular coordinate = $X/R$
$\lambda$	film heat transfer coefficient
$\mu_0$	permeability of free space or air $\mu_0 = 4\pi \times 10^{-7}$ AT/m
$\omega$	angular speed
$\xi$	dimensionless molecular viscosity (= $\eta / \eta_i$ )
$\rho$	lubricant density